Ling 409   Homework 24 for Chapter 16, parts 16.1, 16.2

1. Take the grammar in (16-4). Write out derivations of the following strings (there may be more than one right answer for each):
a. ababab
b. baba
c. baab

2. Write a grammar that generates the following set of strings:
{a, ba, bba, bbba, bbbba, ...}.

3. Study the definition of the reversal operation x^R on pp 432-433. Then prove by mathematical induction that for all strings x,y on a given alphabet A, $(x \frown y)^R = x^R \frown y^R$.

Notes: (i) You are free to choose a specific alphabet A if that makes it easier; just make sure your alphabet contains at least two different symbols. But it's probably just as easy to assume an arbitrary alphabet A.

(ii) The tricky part is how to set up the induction, when there are two different strings x and y mentioned in the problem. I think the easiest way is to leave x arbitrary and to use induction on the length of y. So the way to state the theorem you're trying to prove as a property of n would be this:

Given an alphabet A, then for all strings x on A and all strings y of any length n on A, $(x \frown y)^R = x^R \frown y^R$.

[This is equivalent to the original statement; if we have all strings y of any length n, then we simply have all strings y. Putting in the n is just a guide to how to do this by induction.]

Start with y of length 0 for the basis step. That means that the first thing to prove is: For all strings x on A and all strings y of length 0 on A, $(x \frown y)^R = x^R \frown y^R$. Use the fact that the only string of length 0 is the empty string e.

When you work on the induction step, you will want to use part (2) of definition 16.1.