Homework 5: pp.51-3, #4,5

Answers

(4) Faulty reasoning:
A relation R is reflexive if for all x∈dom(R), <x,x>∈R.
The reasoning does not allow for the case that there is some x∈dom(R) s.t. for all y∈range(R), <x,y>∉R (i.e. *x* bears no relation in R).
For example,
-S is the set of humans
- R is a relation defined on S such that aRb iff a and b have the same parents and those parents have at least two children.
For an only child *x*, *x* does not bear any relation to any other human, including him/herself. Therefore R is not reflexive, though it is symmetric and transitive.

(5)
(a) R = {<1,1>, <2,2>, <3,3>, <5,5>, <6,6>, <10,10>, <15,15>, <30,30>,
<1,2>, <1,3>, <1,5>, <1,6>, <1,10>, <1,15>, <1,30>,
<2,6>, <2,10>, <2,30>,
<3,6>, <3,15>, <3,30>,
<5,10>, <5,15>, <5, 30>,
<6,30>, <10,30>, <15,30>}

- R is a weak order if it is reflexive and antisymmetric.
  (i) R is reflexive: for all x∈dom(R), <x,y>∈R
  (ii) R is antisymmetric: for all <x,y>∈R where x≠y, <y,x>∈R.

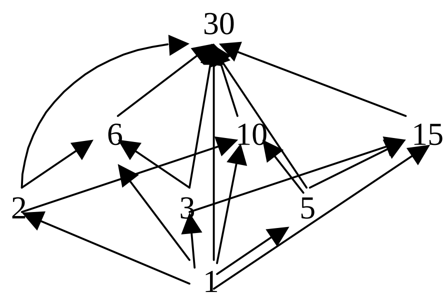
- R is a partial order if it is unconnected.
  (i) R is connected if for every two distinct x,y, <x,y>∈R or <y,x>∈R.
  <3,10>∉R, <10,3>∉R.

(b) 1 is minimal and least.
- No element precedes 1.
- 1 precedes all other elements.

30 is maximal and greatest.
- no element follows 30
- 30 follows all other elements.

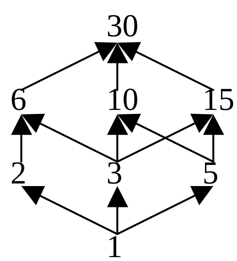
PTO for diagrams…

Here is a *predecessor diagram*.



! Also add loops from each element to itself !

Note: the book actually asked for an *immediate predecessor* diagram. Such a diagram looks like this:



Immediate predecessors of *x* are all *y* s.t. there is no *z* s.t. $y<z<x$.
For example, 1 is not an immediate predecessor of 30 because $1<10<30$.
Note that no element can be an immediate predecessor of itself.

(c) Do the same for the set $\wp$({a,b,c}) and the relation “is a subset of”.

(i) $\wp$({a,b,c}) = {∅, {a}, {b}, {c}, {a,c}, {a,b}, {b,c}, {a,b,c}}
(ii) R={<∅, ∅>, <∅, {a}>, <∅, {b}>, <∅, {c}>, <∅, {a,c}>, <∅, {a,b}>, <∅, {b,c}>, <∅, {a,b,c}>,
<{a}, {a}>, <{a},{a,c}>, <{a},{a,b}>, <{a},{a,b,c}>,
<{b}, {b}>, <{b},{b,c}>, <{b},{a,b}>, <{b},{a,b,c}>,
<{c}, {c}>, <{c},{a,c}>, <{c},{a,c}>, <{c},{a,b,c}>,
<{a,b}, {a,b}>, <{a,b},{a,b,c}>,
<{a,c}, {a,c}>, <{a,c},{a,b,c}>,
<{b,c}, {b,c}>, <{b,c},{a,b,c}>,
<{a,b,c},{a,b,c}>}

- R is a weak order if it is reflexive and antisymmetric.
  (i) R is reflexive: for all x∈dom(R), <x,y>∈R
  (ii) R is antisymmetric: for all <x,y>∈R where x≠y, <y,x>∈R.

- R is a partial order if it is unconnected.

    (i) R is connected if for every two distinct x,y, $\langle x,y \rangle \in R$ or $\langle y,x \rangle \in R$.

    $\langle \{b\}, \{a,c\} \rangle \notin R$, $\langle \{a,c\},\{b\} \rangle \notin R$.