pp.129-134, 1-3, 4a-c, 5a-c, 6a-d, 7, 8a-c
Wednesday, October 03, 2001

Many of the answers are in the back of the book. Here I provide working for (most of) them, plus comments on how to approach them.

(3a) ((p&q) & s)
= (T&T)&F
= T & F
= F

(3c) $p \rightarrow s$
= $T \rightarrow F$
= F

(3e) $(p\&q) \leftrightarrow (r \& \sim s)$
= $(T\&T) \leftrightarrow (T \& \sim F)$
= $(T\&T) \leftrightarrow (T \& T)$
= $T \leftrightarrow T$
= T

(4) (a) and (b) are logically equivalent…
(4a)

p	q	$p \vee \neg q$
1	1	1
1	0	1
0	1	0
0	0	1

(4b)

p	q	$\neg (\neg p \& q)$
1	1	1
1	0	1
0	1	0
0	0	1

(4c)

p	q	$(p \leftrightarrow q) \& p)$
1	1	1
1	0	0
0	1	0
0	0	0

(4d)

p	q	r	$(p \to (q \vee \neg r)$
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

(4e)

p	q	$(((p \to q) \to p) \to q)$
1	1	1
1	0	0
0	1	1
0	0	1

(5) What values of p & q make the statements false?
(a) $p \vee q = 0$ if p=0 and q=0
(b) $((p \vee q) \to (p \,\&\, q)) = 0$ if p=1 and q=0 or if p=0 and q=1.
(c) $\neg (\neg q \vee p) \vee (p \to q) = 0$ if p=1 and q=0.

(6)
(a) $(p \vee \neg p)$ is a tautology. How do we know?
Let's say that there is some value for p s.t. $(p \vee \neg p)=0$.
For $(p \vee \neg p)$ to be 0, both arguments of $\vee$ must be 0.
So p=0 and $\neg p=0$.
If $\neg p=0$, though, p must be 1.
Contradiction.
Therefore all values of $(p \vee \neg p)=1$.

(b) $p \vee q$ is a contingency.
There is some value of p,q (i.e. at least one =1) where $p \vee q=1$ and other values (i.e. both 0) where $p \vee q=0$.

(c) $((p\&q) \to (p \vee r))$ is a tautology.
Let's try to make $((p\&q) \to (p \vee r))=0$:
$((p\&q) \to (p \vee r))=0$
  $1 \quad \to \quad 0 \quad =0$
  $1 \quad \to (0 \vee 0)=0$, therefore p=0 and r=0, therefore:
$((0\&q) \to (0 \vee 0)) = 0$
There is no value of q s.t. 0&q=1.

Therefore, all values must=1.

(d) (¬p & ¬(p→q)) is a contradiction.
Does this ever equal 1?
(¬p & ¬(p→q))=1
1 & 1 =1
¬0 & ¬0 = 1 therefore p = 0, so
¬0 & ¬(0→q) = 1
We need (0→q) to equal 0. But there is no value of q s.t. 0→q=0.
Therefore all values of (¬p & ¬(p→q)) must equal 1.

(7) (a)
To deal with these problems, try drawing up a truth table.

p	q	p→q	p & q	p & ¬q	**¬ (p & ¬q)**
1	1	1	1	0	1
1	0	0	0	1	0
0	1	1	0	0	1
0	0	1	0	0	1

(b) ¬ (¬ p ∨ ¬ q)

(c) (p→q) & (q→p)

(d) Define the connectives in terms of & and ¬.
(1) p ∨ q = ¬ (¬p & ¬q)
(2) p→q = ¬(p & ¬q)
(3) p↔q = ¬(p & ¬q) & ¬(q & ¬p)

(8) These questions are like the set theoretic equalities. Use the same procedures.
(c) ¬p & ((p & q) ∨ (p & r))

= (¬p & (p & q)) ∨ (¬p & (p & r))	Distr.
= ((¬p & p) & q) ∨ (¬p & (p & r))	Assoc.
= (F & q) ∨ (¬p & (p & r))	Compl
= F ∨ (¬p & (p & r))	Identity
= ¬p & (p & r)	Identity
= (¬p & p) & r	Assoc
= F & r	compl
= F	Identity