

Notes on Assignment 18: p243, 1, 2acdfgij, 3, 4, [5e-k was moved into Homework 14]

Notes on problem 1: You have used truth tables to show whether two formulas are equivalent or not. This question was different: you're asked whether the truth of one formula tells you anything about the truth of another. You can do this one either by reasoning in words, or by drawing the two truth tables. If you draw truth tables, then the way to use them for this problem is to look at all the cases (where a "case" is a particular assignment of truth values to the atomic proposition letters) in which the first formula gets a 1 in its final column, and ask whether those cases all get the same truth value in the final column of the second formula. In this case you see that whenever the formula  $p \leftrightarrow q$  gets the value 1, the formula  $p \vee \neg q$  also gets the value 1. (Not vice versa, so they're not equivalent.)

There are other ways to show this, and there were a variety of correct answers on your papers. One other, more 'syntactic', way would be to do a proof of  $p \vee \neg q$  from the premise  $p \leftrightarrow q$ , starting by turning  $p \leftrightarrow q$  into  $(p \rightarrow q) \& (q \rightarrow p)$ , then use Comm and Simp to derive  $(q \rightarrow p)$ , then use the equivalence of that with  $\neg q \vee p$ , then use Comm once more to derive  $p \vee \neg q$ . That also allows you to conclude that whenever  $p \leftrightarrow q$  is true, so is  $p \vee \neg q$ . But the question invited a more semantic argument, and that's what you all gave, which is good.

Notes on problem 2, the proofs.

- Simplification: Sometimes you need to do Comm(utativity) first, since Simp only gets you the FIRST conjunct. From  $p \& q$ , you can derive  $p$  by Simp. If you want  $q$ , then first use Comm to get  $q \& p$ , then Simp to get  $q$  from that.
- Using disjunctive syllogism (D.S.)  
If you have  $\neg p \vee r$  and you have  $p$ , in order to use D.S. you first need to derive  $\neg\neg p$  from  $p$  by Double Negation (one of the Complement Laws). Then you have what you need – a disjunction  $\phi \vee \psi$  and the negation of the first disjunct.
- Conditional proof vs. Hypothetical Syllogism. – Don't mix them; use one OR the other in cases where either one would work, as for instance if you were given the following:

Prove:             $p \rightarrow q$   
                       $q \rightarrow \sim r$   
                       $\sim r \rightarrow s$   
                       $p \rightarrow s$

This one could be done very easily in either of two ways: (i) two applications of H.S., or (ii) Take  $p$  as an auxiliary premise, and do three applications of M.P. and finish up with "Conditional Proof".

But what you don't want to do is to take  $p$  as an auxiliary premise and then use H.S., or you may never manage "discharge" the auxiliary premise. Any proof that puts in an auxiliary premise has to discharge it with an invocation of the rule of Conditional Proof.