Ling 409, Fall 2005 Florian Schwarz

Logistics:

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Feel free to contact me with questions!

Office Hours: Monday, 9/12, 3:45 – 4:45
South College, 317 (Prof. Partee's office)

Let me know if that time doesn't work for you, and we can make an appointment!

Homework: If we forget to list what exercises are no the homework, you can always check on the course website: http://www.people.umass.edu/partee/409/

Review:

- Set: abstract collection of objects (concrete or abstract)
- Identity: same members
- Cardinality: number of members in the set
- Membership vs. Subset relation
- Power sets (where does the name come from?)

See also the 'Cheat Sheet' on the reverse side

Set Theory Cheat Sheet

Operation / Relation & Symbol	**Formal description**	**Informal description**	**Areas to be shaded in Venn-diagram**	**Example**
Union: A ∪ B	{x | x ∈ A **or** x ∈ B}	The set of all objects that are in A or in B (or in both)	1, 2, 3	{a, b, c} ∪ {b, c, d} = {a, b, c, d}
Intersection: A ∩ B	{x | x ∈ A **and x** ∈ B}	The set of all objects that are in A and in B	1	{a, b, c} ∩ {b, c, d} = {b, c}
Difference: A – B	{x | x ∈ A **and** x ∉ B}	The set of a ll objects that are in A but not in B	2	{a, b, c} – {b, c, d} = {a}
Complement: A'	{x | x ∉ A}	The set of all objects that are not in A	3, 4	{a, b, c}' = {d, e} [assuming that U = {a, b, c, d, e}]
Subset Relation: A ⊆ B		All members of A are members of B	True if 2 is empty	{a, b, c} ⊆ {a, b, c, d}
Proper Subset: A ⊂ B		All members of A are members of B and there are members in B that are not in A	True if a) 2 is empty b) 3 is not empty	{a, b, c} ⊂ {a, b, c, d} But: {a, b, c} ⊄ {a, b, c}

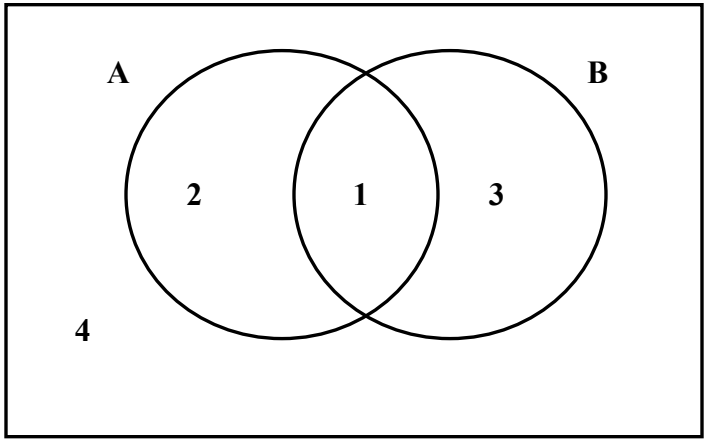
Some basic symbols and what they mean:

x ∈ A: x is a member of the set A
x ∉ A: x is not a member of the set A

∅: the empty set
$\mathscr{P}$(A): The Power Set of A – the set containing all subsets of A

'/': Used in various symbols expressing relations to say that the relation does not hold [e.g. ∉, ⊄, etc.]`

Venn Diagram Schema



1.8. Set-theoretic equalities

There are a number of general laws about sets which follow from the definitions of set-theoretic operations, subsets, etc. A useful selection of these is shown below. They are grouped under their traditional names. These equations hold for any sets X, Y, Z:

1. *Idempotent Laws*
   (a) $X \cup X = X$ (b) $X \cap X = X$

2. *Commutative Laws*
   (a) $X \cup Y = Y \cup X$ (b) $X \cap Y = Y \cap X$

3. *Associative Laws*
   (a) $(X \cup Y) \cup Z = X \cup (Y \cup Z)$ (b) $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

4. *Distributive Laws*
   (a) $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ (b) $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

5. *Identity Laws*
   (a) $X \cup \varnothing = X$ (c) $X \cap \varnothing = \varnothing$
   (b) $X \cup U = U$ (d) $X \cap U = X$

6. *Complement Laws*
   (a) $X \cup X' = U$ (c) $X \cap X' = \varnothing$
   (b) $(X')' = X$ (d) $X - Y = X \cap Y'$

7. *DeMorgan's Laws*
   (a) $(X \cup Y)' = X' \cap Y'$ (b) $(X \cap Y)' = X' \cup Y'$

8. Consistency Principle
   (a) $X \subseteq Y$ iff $X \cup Y = Y$ (b) $X \subseteq Y$ iff $X \cap Y = X$

We will see later that operations on subsets of a set form a *Boolean algebra*.

Another Boolean algebra familiar to you is arithmetics. Some of the laws above also hold there, in particular 2, 3, and 4 (in one direction).

How about the others?
Idempotency does not hold: $x + x \neq x$ (unless $x = 0$)

Identity does hold to some extent: $x + 0 = x$, $x * 0 = 0$, and $x * 1 = 1$
But not: $x + 1 = 1$…

The laws above can be helpful in simplifying expressions denoting sets. Let's simplify the following expression together: $(A \cup B) \cup (B \cap C)'$.